Charakterisierung des Gleichgewichtspunktes $(0,0)$ von

$$\dot{x} = a_{11}x + a_{12}y \qquad \text{mit} \qquad \Delta := \det(a_{ij}) \neq 0,$$
$$\dot{y} = a_{21}x + a_{22}y$$

$$\lambda_{1/2} = \frac{1}{2}\left(a_{11} + a_{22} \pm \sqrt{(a_{11}+a_{22})^2 - 4\Delta}\right)$$

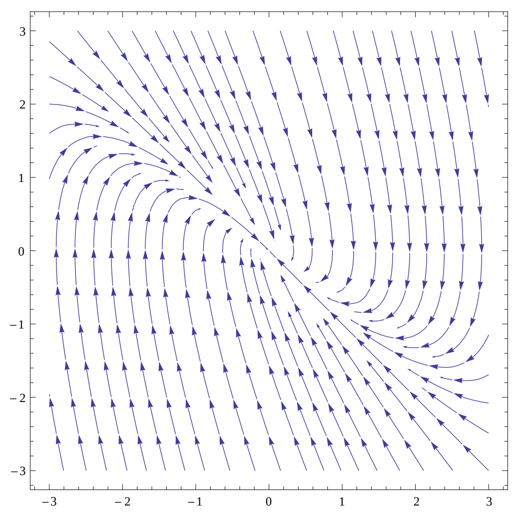
<u>1. Fall:</u> $\lambda_{1/2} \in \mathbb{R}$, $\lambda_1 \neq \lambda_2$, $\lambda_1\lambda_2 > 0$

a) $\lambda_{1/2} < 0$:

(0,0) *stabiler Knotenpunkt*

Beispiel: $\dot{x} = y$
$\dot{y} = -2x - 3y$

StreamPlot[{y, -2 x - 3 y}, {x, -3, 3}, {y, -3, 3}]

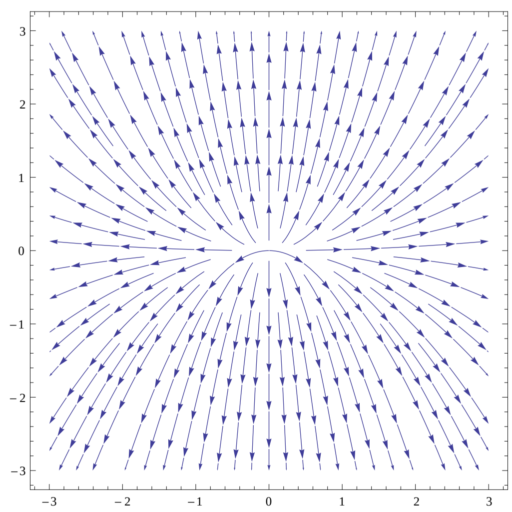


b) $\lambda_{1/2} > 0$:

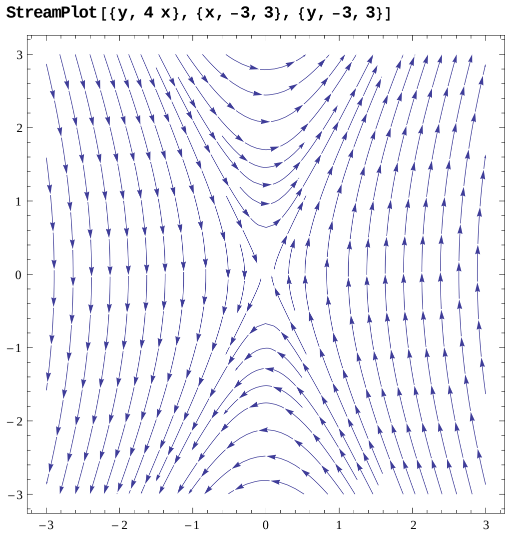
(0,0) *instabiler Knotenpunkt*

Beispiel: $\dot{x} = x$
$\dot{y} = 2y$

StreamPlot[{x, 2 y}, {x, -3, 3}, {y, -3, 3}]



2. Fall: $\lambda_{1/2} \in \mathbb{R}$, $\lambda_1 \lambda_2 < 0$

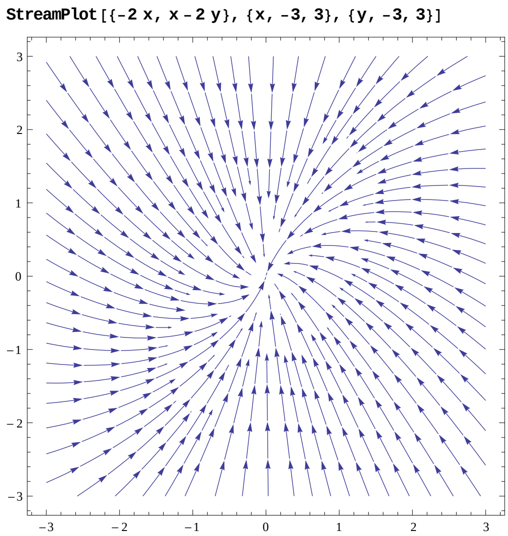


(0,0) *instabiler Sattelpunkt*

Beispiel: $\dot{x} = y$
$\dot{y} = 4x$

3. Fall: $\lambda_1 = \lambda_2 = \lambda \in \mathbb{R}$

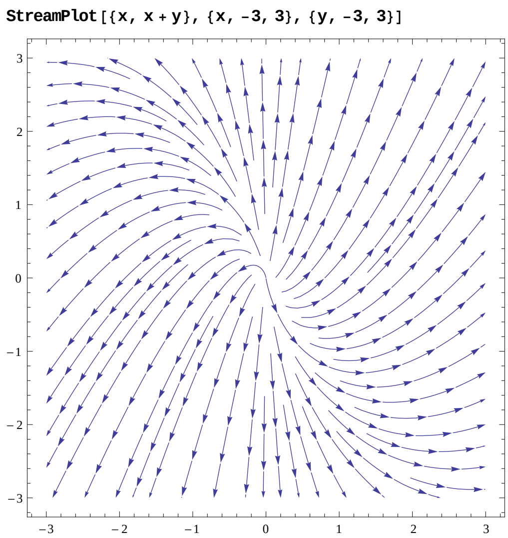
a) $\lambda < 0$:



(0,0) *stabiler Knotenpunkt*

Beispiel: $\dot{x} = -2x$
$\dot{y} = x - 2y$

b) $\lambda > 0$:



(0,0) *instabiler Knotenpunkt*

Beispiel: $\dot{x} = x$
$\dot{y} = x + y$

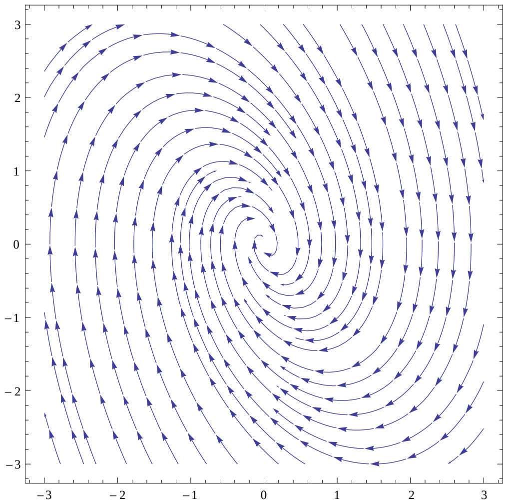
<u>4. Fall:</u> $\lambda_{1/2} \in \mathbb{C},\ \lambda_1 = \overline{\lambda_2},\ \mathrm{Re}\lambda_{1/2} \neq 0$

a) $\mathrm{Re}\lambda_{1/2} < 0$:

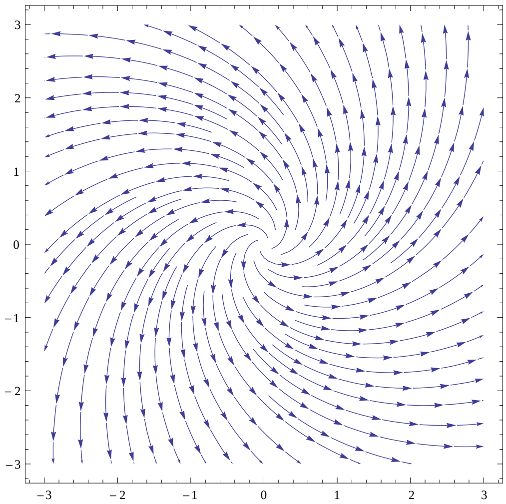
(0,0) *stabiler Strudelpunkt*

Beispiel: $$\dot{x} = y$$
$$\dot{y} = -2x - y$$

StreamPlot[{y, -2 x - y}, {x, -3, 3}, {y, -3, 3}]



StreamPlot[{x - y, x + y}, {x, -3, 3}, {y, -3, 3}]



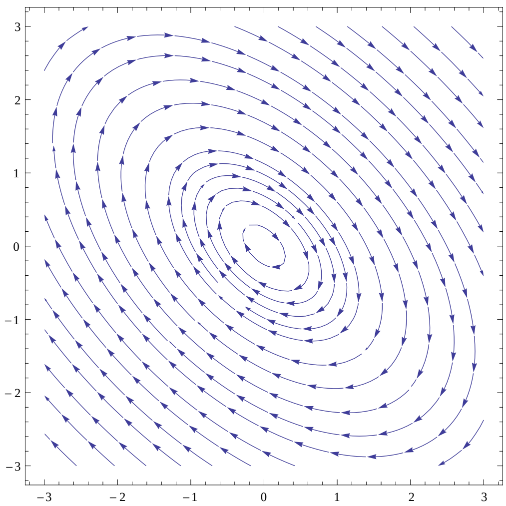
b) $\mathrm{Re}\lambda_{1/2} > 0$:

(0,0) *instabiler Strudelpunkt*

Beispiel: $$\dot{x} = x - y$$
$$\dot{y} = x + y$$

<u>5. Fall:</u> $\lambda_{1/2} \in \mathbb{C},\ \lambda_1 = \overline{\lambda_2},$ $\mathrm{Re}\lambda_{1/2} = 0$

StreamPlot[{x + 2 y, -2 x - y}, {x, -3, 3}, {y, -3, 3}]



(0,0) *Wirbelpunkt, Zentrum*

Beispiel: $$\dot{x} = x + 2y$$
$$\dot{y} = -2x - y$$