Credit Risk Modelling under Incomplete Information and Nonlinear Filtering

Applicant

Prof. Dr. Rüdiger Frey
Mathematisches Institut, Universität Leipzig
Postfach 10 09 20
D-04009 Leipzig
fon: +49 341 9732181
fax: +49 341 9732197
email: ruediger.frey@math.uni-leipzig.de
www: http://www.math.uni-leipzig.de/∼frey

Cooperation partners. We will cooperate with the following foreign partners

1. Prof. Dr. Wolfgang Runggaldier
Dipartimento di Matematica Pura ed Applicata
Università degli Studi di Padova
Italy

2. Quantitative Methods Team, FitchRatings; responsible contact:
Dr. Ahmet E. Kocagil, Managing Director
One State Street Plaza
New York
NY 10004, USA

Summary

The project is concerned with the development of realistic and tractable dynamic credit risk models for the valuation and the hedging of credit derivatives such as corporate bonds or related portfolio products. A new information-based approach mimicking the informational advantages of institutional investors is proposed. Under this approach, this informational advantage is reflected in prices of traded credit derivatives; secondary-market investors will therefore try to back out this information from observed prices. Our project will study the optimal solution of this problem, using advanced methods from stochastic calculus and in particular from nonlinear filtering theory. Moreover, we plan to analyze the ensuing *dynamics* of prices of traded securities and credit spreads. Further research goals include calibration and testing of the model using market data; the development of suitable hedging strategies under incomplete information; the extension of our approach to equity-derivatives markets;

and finally the analysis of portfolio-optimization problems in our context, with particular focus on incomplete information.

1 State of the art and preliminary work

1.1 State of the art

In recent years the market for credit derivatives has witnessed a tremendous growth; in particular, there is now a very liquid market for credit default swaps (CDSs) and for portfolio products such as collaterized debt obligations (CDOs). This growth was accompanied by intensive developments on the regulatory side such as the Basel II process. In response, credit risk modelling has become a very active research area in academia and financial industry, with a particular focus on dynamic models for the pricing and the risk management of portfolio products.

Credit risk modelling is a challenging task since a suitable model should ideally meet a number of demanding criteria:

- The model has to be in line with the basic principles of financial mathematics such as absence of arbitrage.
- The model should lead to *realistic dynamics* of credit spreads (the difference in the yield of a product subject to bankruptcy or default risk and a default-free product). Ideally the model should allow for *contagion* (the fact that credit spreads sometimes jump in reaction to defaults of other firms) and for *spread risk* (random fluctuations of credit spreads between defaults).
- In credit portfolio modelling it is crucial to model the *dependence structure* of default times in an appropriate way; in particular, the model should allow for infrequent clusters of defaults. It is well-known that this is necessary to explain observed prices of traded portfolio products and in particular the *implied correlation skew* of CDO prices (see for instance Hull & White (2004)).
- The model needs to be *tractable*, i.e. the calibration of model parameters to market data and the computation of prices and hedge ratios should be possible with a reasonable computational effort.

Existing credit risk models. Credit risk models can be divided into *firm-value models* on the one hand and *reduced-form models* on the other. In firm-value models default occurs whenever a stochastic process V, generally representing an asset value, falls below a threshold, generally representing liabilities. In reduced-form models the precise mechanism leading to default is left unspecified; the default time of a firm is modelled directly by a nonnegative random variable whose distribution typically depends on economic covariables.

In practice reduced-form pricing models are usually preferred for tractability reasons. Broadly speaking, reduced-form credit portfolio models can be divided in three different classes: models with conditionally independent defaults; factor copula models; models with direct interaction between default intensities. In models with conditionally independent defaults such as Duffie & Garleanu (2001) or Graziano & Rogers (2006), default times are assumed to be independent given the realization of some *observable* economic factor process X. This makes the models easy to treat, as the valuation results of Lando (1998) carry over to the portfolio case; moreover, fluctuations in X lead to fluctuations in default intensities and credit spreads. On the downside, in this model class there is no default contagion; moreover, it may be difficult to obtain realistic levels of default correlation, at least if X follows a diffusion process.

Factor copula models such as Li (2001), Hull & White (2004), Laurent & Gregory (2005) or Kalemanova, Schmid & Werner (2005) have become the market standard for the pricing of CDOs. In a nutshell, in these models one starts from assumptions on the risk neutral distribution function of the default times under consideration. This distribution function can be decomposed into marginal distributions and dependence structure (copula). Marginal distributions are determined by calibrating the model to defaultable-term-structure data, the copula function is specified by the modeller. Usually the copula has a latent factor structure with *unobservable* common factor variable X. The separation between marginal distribution and dependence structure facilitates the calibration of the model. This is the main reason for the popularity of this model class. However, factor copula models are usually presented and used in a *static* fashion, i.e. with a focus on the distribution function of the default times. This makes it hard to to derive model-based hedging strategies and to gain intuition for dynamic aspects of the model.

Finally, in models with interacting intensities such as Jarrow & Yu (2001), Davis & Lo (2001), Giesecke & Weber (2006) or Frey & Backhaus (2006) default intensities are taken as model primitives; in particular, the modeller specifies explicitly the impact of the default of a firm on the default intensities of the remaining firms. This allows for a very intuitive parameterization of dependence between defaults; moreover, the dynamic nature of the model permits the derivation of model-based hedging strategies. On the downside, in models with interacting intensities the calibration to defaultable-term-structure data is usually a computer-intensive procedure, restricting the scope of this model class.

Incomplete information and nonlinear filtering. In most of the above models the distribution of default times is driven by a state variable (process) X. In practical implementations X is usually not associated with observable economic quantities but is treated as latent (unobservable) process. This has far-reaching implications for the dynamics of credit spreads generated by the models:

- With unobservable state variables one frequently encounters the phenomenon of *information-driven default contagion*: given that, say, firm j defaults at time t, the conditional distribution of X_t is updated, leading to a jump (typically upward) in the default intensity of firms $i \neq j$; see for instance Schönbucher (2004) or Section 9.8 of McNeil, Frey & Embrechts (2005) for a discussion in the context of factor copula models.
- It is well-known that in firm value models with observable continuous asset value process V and observable default barrier, the default time is a predictable stopping time so that default intensities do not exist. Kusuoka (1999) and Duffie & Lando (2001) noted that the picture changes if investors have only noisy information of V. In particular, incomplete information on V leads to a more realistic behaviour of short-term credit

spreads.

This calls for a systematic analysis of credit risk models with incomplete information on state variables. A first step in this direction was made in Frey & Runggaldier (2006); in this paper it was shown that in reduced form models where default intensities are driven by an *unobservable*[1] factor process X, the pricing of credit derivatives leads to a nonlinear filtering problem in a natural way. This filtering problem is then studied in detail in a very general Markovian model where the factor process may jump at default times; in particular the paper derives a finite-dimensional filter for the case where X follows a finite state Markov chain and provides a filter-approximation result. The relation between derivative pricing and nonlinear filtering is also discussed in Gombani, Jaschke & Runggaldier (2005) (for default-free term structure models) and in Frey & Schmidt (2006) (for structural credit risk models under noisy information of the asset value). Blanchet-Scalliet & Jeanblanc (2004) take a more general look on the role of information in credit risk modelling. Related - but technically much simpler - work on from the mathematical finance area includes Collin-Dufresne, Goldstein & Helwege (2003), Giesecke (2004) and Schönbucher (2004)

In the present project it is intended to extend the analysis of Frey & Runggaldier (2006) and to focus on the implications of incomplete information about X for the *dynamics* of credit spreads. We will use a new modelling approach - explained in detail in Section 3.2 - which is inspired by informational aspects of the price-formation process on real markets. We are optimistic that the new approach permits us to address some of the challenges mentioned above. The mathematical tools stem mainly from stochastic calculus and in particular from nonlinear filtering.

Nonlinear filtering methodology. The general theory of nonlinear filtering is well-developed; see for instance the excellent overview in Darling (2004) and the references given therein. Recent years have also brought significant progress in the development of numerical, especially simulation-based tools such as MCMC (Markov Chain Monte Carlo) methods or particle filtering, substantially extending the range-of-applicability of filtering methods. Filtering techniques are increasingly being used in financial mathematics. Particularly relevant for the present project are the papers Schweizer (1994), Frey (2000) or Ceci & Gerardi (2006) (for hedging under incomplete information); Frey & Runggaldier (2001) or Cvitanic, Liptser & Rozovski (2006) (for filtering with marked-point-process information); Sass & Haussmann (2004) is a nice example for the work on portfolio optimization under incomplete information.

1.2 Preliminary work

Over the last years, credit risk modelling has been a major research topic at the financial-mathematics working group at the University of Leipzig, headed by Prof. R. Frey and JProf T. Schmidt. This has led among others to the following publications: static portfolio credit risk models are considered in Frey & McNeil (2003); the state of the art in dynamic credit risk modelling is summarized in Chapter 9 of McNeil et al. (2005); Frey & Backhaus (2006) is concerned with portfolio models with interacting intensities and contains preliminary results

[1] In Frey & Runggaldier (2006) investor information consists of the default history of the portfolio and of observations of X in additive Gaussian noise; these might be interpreted as noisily observed credit spreads

on dynamic hedging of credit derivatives; First results regarding the link between nonlinear filtering and credit risk are contained in Frey & Runggaldier (2006) (for portfolio credit risk models) and in Frey & Schmidt (2006) (for structural models with incomplete information on the asset value); however, in these works the dynamics of the resulting security prices are not considered. Frey (2000) and Frey & Runggaldier (2001) use nonlinear filtering techniques to analyze security market models with incomplete observations suitable for high-frequency stock-return data.

2 Goals and Work Schedule

2.1 Goals

We propose to study a new, information-based approach to modelling the dynamics of credit spreads of a given portfolio of firms using theoretical results and numerical techniques from nonlinear filtering. Our research agenda contains the following items: analyze theoretical and computational aspects related to the pricing and the hedging of credit derivatives; study the ensuing nonlinear filtering problems; test and implement the model with simulated and real data; extend the modelling approach to other markets and model classes, in particular to structural credit risk models with incomplete information on the asset value; study portfolio optimization problems in this new framework. We are confident that the new model will overcome some of the deficiencies of the static copula-based credit risk models currently in use, thus contributing to the efforts of putting the pricing and hedging of portfolio credit derivatives on a firmer methodological footing.

2.2 Work schedule

Description of the information-based modelling approach

We start with a brief description of our information based modelling approach, as the latter is central to the proposed research agenda. We are interested in modelling the dynamics of the default-state and the credit spreads of a portfolio of m firms; the random time τ_i denotes the default time of firm i, $Y_{t,i} = 1_{\{\tau_i \le t\}}$ is the corresponding default indicator, and $Y_t = (Y_{t,1}, \dots, Y_{t,m})$ gives the current default state. Default intensities will be driven by some factor process X. In our analysis we distinguish three different layers of information: full information; information of informed market participants (market-information); information of secondary-market investors (investor-information):

Full information. The full-information setup is a theoretical device used for the construction of the model. Given some probability space $(\Omega, \mathcal{F}, Q)$, we assume that both X and Y are observable, i.e. the full-information filtration is given by $\mathbb{F} := \mathbb{F}^X \vee \mathbb{F}^Y$. We assume that under full information the τ_i are conditionally independent doubly stochastic random times with $(Q, \mathbb{F})$ default intensity $\lambda_{t,i} = \lambda_i(X_t)$ and that X follows a finite state Markov chain with state space $S^X = \{x_1, \dots, x_{|S^X|}\}$. This setup is akin to the model of Graziano & Rogers (2006). Denote by $r(t) \ge 0$ the (deterministic) risk-free short rate and assume that Q represents a risk-neutral measure. We define the *full-information value* of a $\mathcal{F}_T^Y$-measurable claim H (such as a typical credit derivative) by

$$H_t = E^Q\left(e^{-\int_t^T r(s)ds} H \mid \mathcal{F}_t\right) =: h(t, X_t, Y_t)\,, \tag{1}$$

where the last definition makes sense since the pair (X, Y) is Markov w.r.t. $\mathbb{F}$. Note that full-information values are easy to compute due to the simple structure of the full-information model; see for instance Graziano & Rogers (2006) for details. We emphasize at this point that the process X is unobservable to all investors in the model, so that the model-structure is more sophisticated than merely assuming conditionally independent defaults.

Market-information. The prices of traded credit derivatives are determined by *informed market-participants*. These investors have access to so-called *market information*, given by the filtration $\mathbb{F}^{\mathcal{M}} := \mathbb{F}^Y \vee \mathbb{F}^Z$. The stochastic process Z represents noisy observations of X and can be viewed as abstract form of 'insider information'. Mathematically, Z is modelled as solution of the SDE $dZ_t = a(X_t)dt + dW_t$, W a Brownian motion independent of X (observations of X in additive Gaussian noise). Recall that Q is our pricing measure. The market price of a traded security with payoff H and maturity T is defined as

$$\hat{H}_t := E^Q\left(e^{-\int_t^T r(s)ds} H \mid \mathcal{F}_t^{\mathcal{M}}\right) = E^Q(h(t, X_t, Y_t) \mid \mathcal{F}_t^{\mathcal{M}}) \tag{2}$$

(using (1) and iterated conditional expectations). Since Y_t is known, in order to compute $\hat{H}_t$ one needs to determine the conditional distribution of X_t given $\mathcal{F}_t^{\mathcal{M}}$, represented by the conditional probabilities

$$\pi_t^{\mathcal{M}}(i) = Q(X_t = x_i \mid \mathcal{F}_t^{\mathcal{M}}), \quad 1 \le i \le |S^X|\,.$$

This is a typical nonlinear filtering problem which has been solved for instance in Frey & Runggaldier (2006). The dynamics of $\hat{H}_t$ can be derived by martingale representation arguments similar to the derivation of the Kushner-Stratonovich equation in the innovations approach to nonlinear filtering; first results are documented in Frey & Gabih (2007).

Investor-information. The process Z is used to model the informational advantage of the informed market participants. As will be explained below, this leads to realistic dynamics of credit spreads. However, in our approach the process Z is not directly related to observable economic quantities. We therefore propose to study pricing and hedging of non-traded credit derivatives from the viewpoint of secondary-market investors with information set $\mathbb{F}^I \subset \mathbb{F}^{\mathcal{M}}$ (investor information). We assume that $\mathbb{F}^I$ contains the default history $\mathbb{F}^Y$ and (noisily observed) prices of traded credit derivatives. By a similar argument as in (2), pricing with respect to $\mathbb{F}^I$ leads to a second filtering problem: one has to determine the conditional distribution of X_t given investor information $\mathcal{F}_t^I$. This will typically be a high-dimensional problem, the analysis of which forms a core part of our research agenda.

Advantages. This modelling approach has a number of advantages. Under our approach prices are weighted averages of full-information values (by (2)); hence actual computations are done mostly in the context of the full-information model. Since the latter has a very simple structure, computations become relatively straightforward. They can be based on analytic techniques (using the Kolomogorov equations for the Markov chain X), on simulation techniques, or on Laplace transforms as in Graziano & Rogers (2006). On the other hand, the fact that prices of traded securities are given by the projection of their full-information value on the market filtration $\mathbb{F}^{\mathcal{M}}$ leads to interesting credit-spread dynamics: the proposed approach accommodates *spread risk* (as credit spreads fluctuate in response to fluctuations in Z) and *default contagion* (as defaults of firms in the portfolio lead to an update of the conditional distribution of X given $\mathcal{F}_t^{\mathcal{M}}$ and hence to a jump in the $(Q, \mathbb{F}^{\mathcal{M}})$-default intensities given by $\hat{\lambda}_{t,i} = E^Q(\lambda_i(X_t) \mid \mathcal{F}_t^{\mathcal{M}})$. This feature is important for the derivation of realistic dynamic hedging strategies. Moreover, the model has a natural factor structure with factors given by the conditional probabilities $\pi_t^{\mathcal{M}}(i)$, $1 \leq i \leq |S^X|$. Finally, we expect that the model is easy to calibrate to observed market prices, as the introduction of the conditional probability vector $\boldsymbol{\pi}_t^{\mathcal{M}}$ gives a lot of flexibility: these probabilities can be set by the model-user in response to the available price and default information. Implementation and testing of various model-calibration strategies is part of our research agenda.

Research agenda: short- and medium term goals

The first phase of the project (year one and parts of the second year) will be devoted to the analysis of various aspects of the information-based model sketched above. We will study the following issues.

Hedging of credit derivatives. A very important issue will be the hedging of OTC credit derivatives (for instance protection-seller positions in CDO-tranches) by dynamic trading in liquid single-name products such as CDSs. This topic is currently receiving much attention; see for instance Laurent, Cousin & Fermanian (2007), Bielecki, Jeanblanc & Rutkowski (2004) or our own work Frey & Backhaus (2006). The proposed model is well-suited for this analysis, as it is able to generate a rich set of credit-spread dynamics. We are particularly interested in hedging strategies for secondary-market investors, that is $\mathbb{F}^I$-predictable strategies. As a starting point will use the concept of *risk-minimization with restricted information* put forward by Schweizer (1994) and Di Masi, Platen & Runggaldier (1995). This approach offers two advantages: it allows for market incompleteness, and it permits us to compute $\mathbb{F}^I$-predictable strategies by suitably projecting the integrands in the $\mathbb{F}^{\mathcal{M}}$-Kunita-Watanabe decomposition of a contingent claim onto $\mathbb{F}^I$, thereby helping to deal with the issue of incomplete information.

Filtering for secondary-market investors. Both for pricing and for hedging purposes secondary-market investors need to determine the conditional distribution of X_t given $\mathcal{F}_t^I$, summarized by the conditional probabilities

$$\pi_t^I(i) = Q(X_t = x_i \mid \mathcal{F}_t^I) = E(\pi_t^{\mathcal{M}}(i) \mid \mathcal{F}_t^I), \quad 1 \leq i \leq |S^X| .$$

If one assumes that secondary-market investors observe the prices of traded securities exactly and if the number of traded credit products is large relative to $|S^X|$, the probabilities $\boldsymbol{\pi}_t^{\mathcal{M}}$ used by the market can be determined by ordinary calibration, i.e. by minimizing the sum of (weighted) squared deviations between market- and model prices; note that in that case $\boldsymbol{\pi}_t^{\mathcal{M}} = \boldsymbol{\pi}_t^I$. However, we are mostly interested in the realistic scenario where prices are observed with a (small) amount of noise or where S^X is large. In that case the determination of $\boldsymbol{\pi}_t^I$

becomes a challenging nonlinear filtering problem with $|S^X|$-dimensional signal process $\boldsymbol{\pi}_t^{\mathcal{M}}$ and observations given by the noisily observed credit spreads. This problem will be studied in cooperation with Wolfgang Runggaldier (University of Padova) who is an internationally recognized expert on nonlinear filtering. It is intended to use simulation-based methods from particle filtering citebib:crisan-lyons-99 for tackling this problem numerically.

Numerical implementation and empirical testing. Several variants of the model will be implemented numerically. As a prelude to systematic calibration a simulation study will be performed in order to understand the impact of various model parameters on CDO prices and credit spread dynamics generated by the model. In a second step the model will be calibrated to/tested with real market data such as time series of CDS spreads and spreads of synthetic CDO tranches. We intend to implement a simple calibration method and the more sophisticated filtering approach as outlined above. The necessary data will be provided by the rating agency Fitch. Moreover, we expect that discussions with market practitioners at FitchRatings will be very valuable in identifying and correcting practical deficiencies of the model.

Longer-term goals

In the second phase of the project (years two and three) we plan to study the following topics; details will of course depend on the progress made in Phase 1.

Firm-value models. The information-based approach can be fruitfully extended to other markets. The extension to structural credit risk models with noisily-observed asset-value process V - as in Duffie & Lando (2001), Coculescu, Geman, & Jeanblanc (2006) or Frey & Schmidt (2006) - appears particularly promising. Traded securities include equity and debt. The information-hierarchy is modelled as follows: the asset-value V is the state variable and will be unobservable to all investors. The price of equity (the stock price) is set by by informed market participants (insiders) receiving a noisy observation Z related to V. Secondary-market investors on the other hand observe just prices of traded securities, in particular the stock price of the firm. This distinction between different levels of information is similar to the discussion in Jarrow & Protter (2004). As shown by Frey & Schmidt (2006), in this framework the pricing of corporate securities leads to an interesting nonlinear filtering problem; see also Nakagawa (2001).

As a first step we plan to determine the *dynamics* of the stock price in this context, using arguments from the innovations approach to nonlinear filtering. Next we plan to study the pricing and the hedging of corporate securities from the viewpoint of secondary-market investors. Hybrid products with debt and equity components such as convertible bonds or so-called equity default swaps will be particularly relevant here; see Linetsky (2006) or Bielecki, Crépey, Jeanblanc & Rutkowski (2006) for existing related work. Note that this will naturally lead to infinite-dimensional models as discussed below, since the state variable V (the asset value process) is typically modelled by a diffusion process and has therefore an infinite state space. In the long run we want to study the well-known skew-pattern of implied-volatility equity options from the perspective of our incomplete-information approach.

Infinite-dimensional models. It is natural to extend the information-based approach to models where the unobservable factor X has an infinite state space. For instance, the full-information model could be an affine jump diffusion model as in Duffie & Garleanu (2001). In that case there exists no finite-dimensional filter for determining the conditional distribution of X_t given $\mathcal{F}_t^{\mathcal{M}}$, and suitable filter approximations need to be derived. The resulting credit-spread dynamics (derived as before by projecting the full-information model onto $\mathbb{F}^{\mathcal{M}}$) has then an infinite-dimensional factor structure, leading to interesting connections to SDEs on Hilbert spaces. We intend to explore these connections further.

Portfolio optimization. Finally, we intend to study portfolio-optimization problems (for instance optimal investment in corporate bonds) for credit portfolios. For this we need to extend our setting and specify the asset-price dynamics simultaneously under the historical measure P and the risk-neutral measure Q. This leads to the difficult problem of estimating risk premia for default risk as in Berndt, Douglas, Duffie, Ferguson & Schranz (2005); in line with the overall spirit of our project we intend to attack this problem via nonlinear filtering methods. Suitable model-formulations will lead to hidden Markov models (HMMs) for asset returns. We will use the martingale approach (as in Sass & Haussmann (2004)) and dynamic programming methods to deal with portfolio-optimization problems in this context.

3 The research team

3.1 The team at University of Leipzig

Team members:

- Prof. Dr. Rüdiger Frey, full professor (C4), head of the financial-mathematics working group.
- JProf. Dr. Thorsten Schmidt, assistant professor (Juniorprofessor, W1) in the financial-mathematics working group (see his webpage `www.math.uni-leipzig.de/~tschmidt` for details regarding his scientific activities).
- Ms Lin Xu, PhD student holding a fellowship from the International Max Planck Research School (IMPRS) Mathematics in the Sciences.
- Mr Roland Seydel PHD-student IMPRS. Both are working on optimization problems related to credit-risk.

3.2 Cooperations

The project will be carried out in close cooperation with Prof. W. Runggaldier, Unversity of Padova. The applicant has a long and successful cooperation with Prof. Runggaldier as is documented by the list of joint publications.

Moreover, the project team will collaborate with quantitative analysts within the Quantitative Methods Group of FitchRatings, headed by Dr. Ahmet Kocagil. In particular, FitchRatings will provide some of the necessary financial data and will give useful input regarding institutional details of credit markets. The applicant is member of the academic advisory board of FitchRatings which provides the institutional basis for the collaboration. The co-operation with Fitchratings will be on a strictly non-commercial basis, and all results will be made publicly available. It is not intended to develop ‘production-type’ versions of the models for use within Fitchratings or any other financial institution in the context of the proposed project.

Moreover, we plan to have a regular scientific exchange with Prof. Monique Jeanblanc, Université d’ Evry (France) who is an internationally recognized specialist on dynamic credit risk modelling and with Dr. Jörn Sass, RICAM, Linz (Austria) who is a specialist on portfolio optimization under partial optimization.

References

Berndt, A., Douglas, R., Duffie, D., Ferguson, F. & Schranz, D. (2005), ‘Measuring default risk premia from default swap rates and EDF’s’, Preprint, Stanford University.

Bielecki, T., Crépey, S., Jeanblanc, M. & Rutkowski, M. (2006), ‘Arbitrage pricing of defaultable game options and applications to convertibles’, working paper, Université d’ Evry.

Bielecki, T., Jeanblanc, M. & Rutkowski, M. (2004), Hedging of defaultable claims, *in* ‘Paris-Princeton Lectures on Mathematical Finance’, Vol. 1847 of *Springer Lecture Notes in Mathematics*, Springer.

Blanchet-Scalliet, C. & Jeanblanc, M. (2004), ‘Hazard rate for credit risk and hedging defaultable contingent claims’, *Finance and Stochastics* **8**, 145–159.

Ceci, C. & Gerardi, A. (2006), ‘A model for high frequency data under partial information: a filtering approach’, *International Journal of Theoretical and Applied Finance (IJTAF)* pp. 1–22.

Coculescu, D., Geman, H., & Jeanblanc, M. (2006), ‘Valuation of default sensitive claims under imperfect information’, working paper, Université d’ Evry.

Collin-Dufresne, P., Goldstein, R. & Helwege, J. (2003), ‘Is credit event risk priced? modeling contagion via the updating of beliefs’, Preprint, Carnegie Mellon University.

Cvitanic, J., Liptser, R. & Rozovski, B. (2006), ‘A filtering approach to tracking volatility from prices observed at random times’, *The Annals of Applied Probability* **16**, 1633–1652.

Darling, R. (2004), ‘Nonlinear filtering - online survey’, Online survey, available from `http://www.nonlinearfiltering.webhop.net/` .

Davis, M. & Lo, V. (2001), ‘Infectious defaults’, *Quant. Finance* **1**, 382–387.

Di Masi, G., Platen, E. & Runggaldier, W. (1995), Hedging of options under discrete observation on assets with stochastic volatility, *in* E. Bolthausen, M. Dazzi & F. Russo, eds, ‘Seminar on Stochastic Analysis, Random Fields and Applications’, Vol. 36 of *Birkhäuser Progress in Probability*, pp. 359–364.

Duffie, D. & Garleanu, N. (2001), 'Risk and valuation of collateralized debt obligations', *Financial Analyst's Journal* **57**(1), 41–59.

Duffie, D. & Lando, D. (2001), 'Term structure of credit risk with incomplete accounting observations', *Econometrica* **69**, 633–664.

Frey, R. (2000), 'Risk-minimization with incomplete information in a model for high frequency data', *Math. Finance* **10**(2), 215–225.

Frey, R. and Schmidt, T. & Gabih, A. (2007), 'Pricing and hedging of credit derivatives via nonlinear filtering', preprint, Universität Leipzig, submitted.

Frey, R. & Backhaus, J. (2006), 'Credit derivatives in models with interacting default intensities: a Markovian approach', Preprint, Universität Leipzig. available from `www.math.uni-leipzig.de/~frey`.

Frey, R. & McNeil, A. (2003), 'Dependent defaults in models of portfolio credit risk', *J. Risk* **6**(1), 59–92.

Frey, R. & Runggaldier, W. (2001), 'A nonlinear filtering approach to volatility estimation with a view towards high frequency data', *International Journal of Theoretical and Applied Finance* **4**, 199–210.

Frey, R. & Runggaldier, W. (2006), 'Credit risk and incomplete information: a nonlinear filtering approach', preprint, Universität Leipzig.

Frey, R. & Schmidt, T. (2006), 'Pricing corporate securities under noisy asset information', preprint, Universität Leipzig,forthcoming in *Mathematical Finance*.

Giesecke, K. (2004), 'Correlated defaults with incomplete information', *Journal of Banking and Finance* **28**, 1521–1545.

Giesecke, K. & Weber, S. (2006), 'Credit contagion and aggregate losses', *J. Econom. Dynam. Control* **30**, 741–767.

Gombani, A., Jaschke, S. & Runggaldier, W. (2005), 'A filtered no arbitrage model for term structures with noisy data', *Stochastic Processes and Applications* **115**, 381–400.

Graziano, G. & Rogers, C. (2006), 'A dynamic approach to the modelling of correlation credit derivatives using Markov chains', working paper, Statistical Laboratory, University of Cambridge.

Hull, J. & White, A. (2004), 'Valuation of a CDO and a nth to default CDS without Monte Carlo simulation', *Journal of Derivatives* **12**, 8–23.

Jarrow, R. & Protter, P. (2004), 'Structural versus reduced-form models: a new information based perspective', *Journal of Investment management* **2**, 1–10.

Jarrow, R. & Yu, F. (2001), 'Counterparty risk and the pricing of defaultable securities', *J. Finance* **56**, 1765–1799.

Kalemanova, A., Schmid, B. & Werner, R. (2005), 'The normal inverse gaussian distribution for synthetic CDO pricing', working paper, Risklab Germany.

Kusuoka, S. (1999), 'A remark on default risk models', *Adv. Math. Econ.* **1**, 69–81.

Lando, D. (1998), 'Cox processes and credit risky securities', *Rev. Derivatives Res.* **2**, 99–120.

Laurent, J., Cousin, A. & Fermanian, J. (2007), 'Hedging default risk of CDOs in Markovian contagion models', workinp paper, ISFA Actuarial School, Université de Lyon.

Laurent, J. & Gregory, J. (2005), 'Basket default swaps, CDOs and factor copulas', *Journal of Risk* **7**, 103–122.

Li, D. (2001), 'On default correlation: a copula function approach', *J. of Fixed Income* **9**, 43–54.

Linetsky, V. (2006), 'Pricing equity derivatives subject to bankruptcy', *Mathematical Finance* **16**(2), 255–282.

McNeil, A., Frey, R. & Embrechts, P. (2005), *Quantitative Risk Management: Concepts, Techniques and Tools*, Princeton University Press, Princeton, New Jersey.

Nakagawa, H. (2001), 'A filtering model on default risk', *J. Math. Sci. Univ. Tokyo* **8**, 107–142.

Sass, J. & Haussmann, U. (2004), 'Optimizing the terminal wealth under partial information: The drift process as a continuous time markov chain', *Finance and Stochastics* **8**, 553–577.

Schönbucher, P. (2004), 'Information-driven default contagion', Preprint, Department of Mathematics, ETH Zürich.

Schweizer, M. (1994), 'Risk minimizing hedging strategies under restricted information', *Math. Finance* **4**, 327–342.