UNIVERSITÄT BIELEFELD
Faculty of Mathematics

Didaktik der Mathematik
Universität Leipzig

Chance or strategy: The LUPI-game – all three probabilistic approaches in one

Dr. Thomas Krohn (Leipzig University)
Dr. Stefanie Schumacher (Bielefeld University)

ICME 15, Sydney ● TSG 1.6 ● 12th July 2024

The LUPI-Game

What‘s already there?

- Within a group, m persons individually and secretly choose a number between 1 and n.
- The winner is the one whose number is the smallest not chosen by anyone else. The winning number is called "LUPI number“.
- If there is no unique smallest number, nobody wins.
- From a game theory perspective, it belongs to the category of static games with imperfect information. (cf. Riechmann 2013, p. 21, p. 36ff.)
- Various studies with focus on game theory exist on LUPI and/or similar situations, e. g. reverse auctions, minority games,... (Challet & Zhang 1997, 1998, Bottazzi & Devetag 2003, 2007; Chmura & Pitz 2006; Devetag et al. 2011; Linde et al. 2014; Otsubo, Kim 2007; Houba, van der Laan & Veldhuizen 2011; Bernhardsson, Juul et al. 2012)

The LUPI-Game

What's still missing?

- no references to the school context in general
- no implementation in "real" groups such as school classes
- no profound qualitative evaluation of the reasoning categories
- no statements on simulation, problem of chance vs. strategy, risk…

To what extent can the ***three approaches to probability (theoretical, frequentist, subjective)*** *be profitably combined with a simple game such as LUPI with its* ***random and strategic influences*** *and can the* ***central requirements for teaching stochastics*** *be covered?*

LUPI - the 3 approaches to probability

Theoretical (Random)	Frequentist (Random)	Subjective (Strategy)
Calculation probability p that exactly one person out of m is the only person who chooses a number k from 1 to n: $p = \sum_{i=1}^{m} \frac{1}{n} \cdot \left(\frac{n-1}{n}\right)^{m-1}$	Simulation Large number of repeats Use of digital media such as GeoGebra, MS Excel	Playing the game Real-life-situation Discussing the individual arguments in class

LUPI- the random game

Approach I

- Theoretical approach to probability

The probability p that exactly one person out of m is the only person who chooses a number k from 1 to n is

$$p = \sum_{i=1}^{m} \frac{1}{n} \cdot \left(\frac{n-1}{n}\right)^{m-1} = m \cdot \frac{1}{n} \cdot \left(\frac{n-1}{n}\right)^{m-1} = \binom{m}{1} \cdot \left(\frac{1}{n}\right)^{1} \cdot \left(\frac{n-1}{n}\right)^{m-1}.$$

- LUPI number 1 easy to calculate, more difficult for LUPI numbers > 1.
- Basically solvable in secondary school level (reduction to e.g. m=n=2,3; Laplace probability, tree diagram, ...), more effective later (binomial distribution).
- General understanding:
  The larger the number, the smaller the probability that it is a LUPI number.

LUPI- the random game

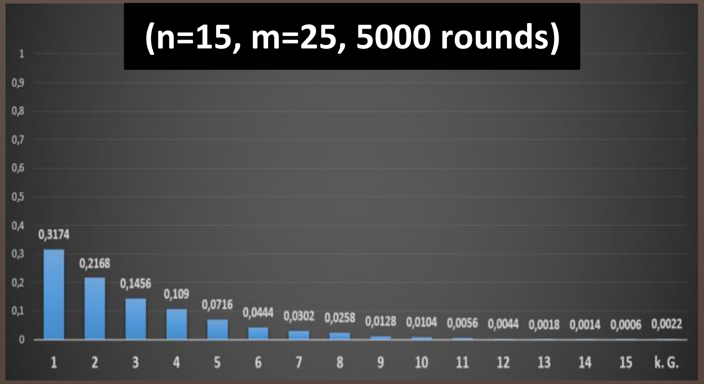
Approach II

- Frequentist approach to probability

Use of digital media such as GeoGebra, MS Excel etc.

Only a few functions required:

- e.g. in Excel: random(), count if()
- large number of independent experiments
- variable parameters n and m
- direct visualization



- **200** students in secondary schools, **4** situations (*n*=15, *m*=1,25,50,100)
- From **grade 9** onwards, the distribution of frequencies was anticipated and justified almost without error.
- **Grades 6 to 8** already assigned and justified > 75% correctly.

Reasoning/communicating: „What happens if ...?“
Simulating is a useful strategy!

LUPI- the strategy game

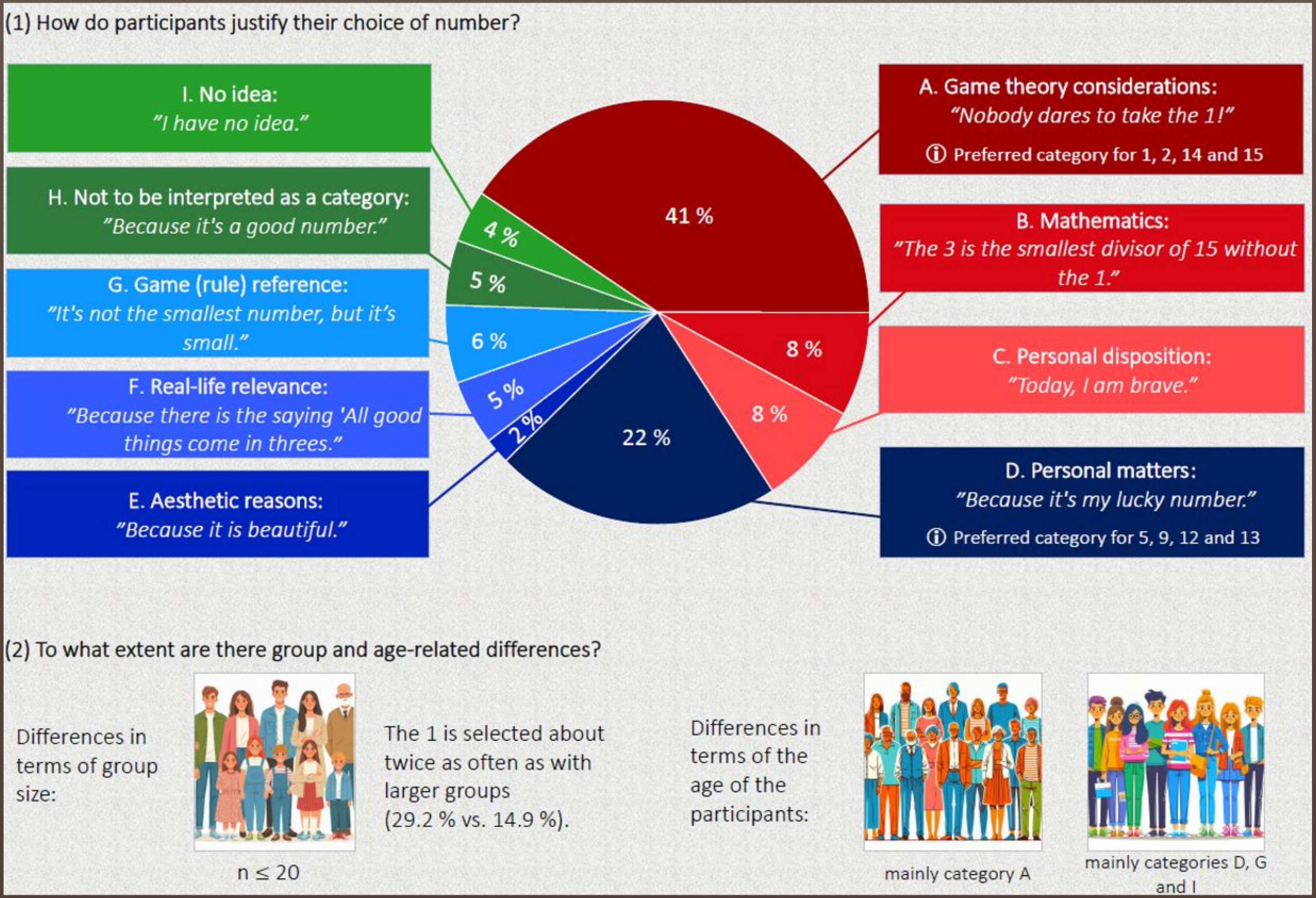
Approach III

- Subjective-strategic approach to probability
  - **29** groups with a total of **656** participants ($\overline{\mathrm{m}} = 22.6; \mathrm{sd} = 6.0$) (06/2024)
  - primary and secondary school learners, student teachers as well as trained teachers and mathematicians from Germany.
  - Participants were given the central rules of the game on a sheet of paper and were asked to give additional reasons (open answer format) for their choice of number 1 to 15.
  - Given explanations were subsequently analyzed by two coders using qualitative content analysis according to Mayring (2015).
  - Interrater reliability (20% of cases, random), Cohen's κ for the nominally scaled categories
    - Code 1 for κ=0.862 ("very good")
    - Code 2 for κ= 0.623 ("good")
    - Code 3 not calculated

LUPI- the strategy game

Approach III

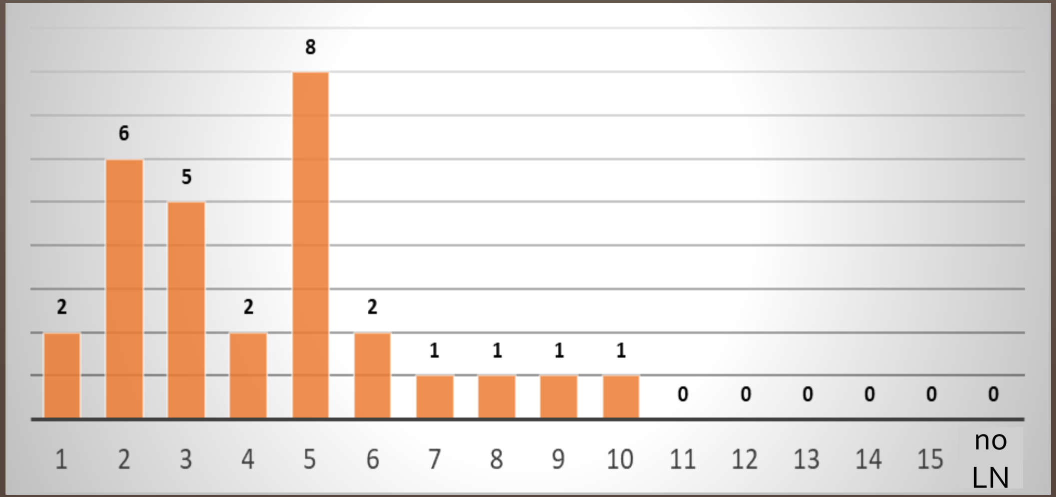
- Subjective-strategic approach to probability



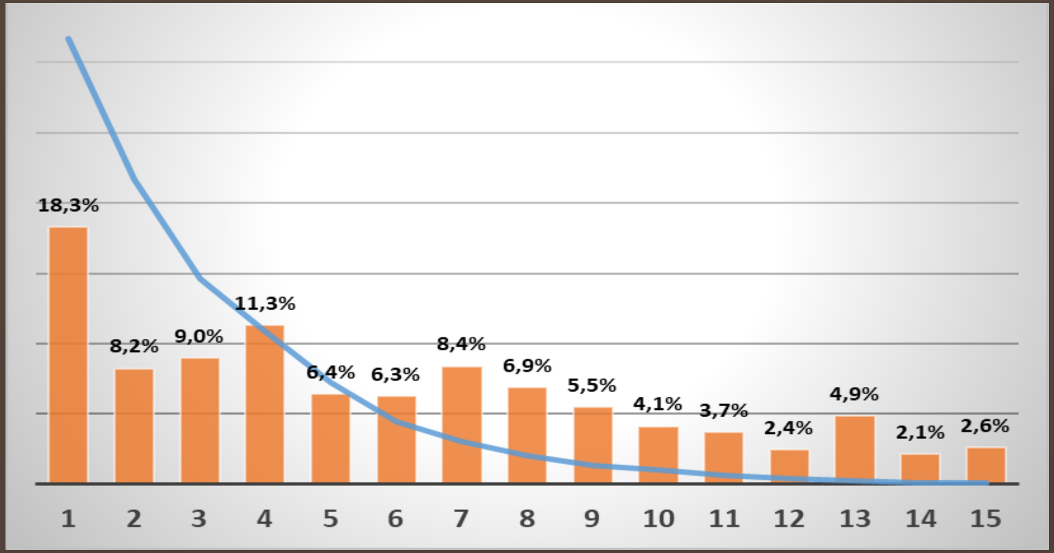
LUPI- the strategy game

Approach III

- Subjective-strategic approach to probability



absolute frequency
of winning numbers



relative frequency
of chosen numbers

LUPI - the 3 approaches to probability

Theoretical (Random)	Frequentist (Random)	Subjective (Strategy)
Calculation $p = \sum_{i=1}^{m} \frac{1}{n} \cdot \left(\frac{n-1}{n}\right)^{m-1}$	Simulation Use of digital media such as GeoGebra, MS Excel etc.	Playing the game Discussing the individual arguments in class
• Easy with LUPI-number 1 • Secondary school knowledge • Effective with use of binomial distribution	• Only a few functions required • Large number of independent experiments • Variable parameters n, m • Immediate visualization	• Motivation and fun • Recurring categories of reasons • Differences in the reasons regarding age & group size • No differences regarding male/female participants

Future studies

- **Learning environments** including simulations with digital media (for schools and high schools)
- **Longitudinal study** including the announcement of distribution of the numbers in between multiple rounds of the game
- „**Risk**“ as an additional focus for analyzing the individual reasons for choosing the number
- **Chance or strategy**: What influences LUPI (more)?

Thank you!

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More information on the LUPI-project

recommended LUPI-, LUBA- & MiGa-references related to game theory

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