Let $V$ be an $n$-dimensional vector space. Let $S : V \otimes V \to V \otimes V$ be the braiding map defined (on simple tensors) by $S(v \otimes w) = w \otimes v$ (see remark 2.11). Let $e_1, \ldots, e_n$ and $\tilde{e}_1, \ldots, \tilde{e}_n$ be bases of $V$ and $e^1, \ldots, e^n$ and $\tilde{e}^1, \ldots, \tilde{e}^n$ the corresponding dual bases of $V^*$. 

**Exercise 1 (3 Points).** Let $h \in V \otimes V$ be a symmetric tensor, i.e. $Sh = h$. Show, that for any $\varphi, \theta \in V^*$, we have 

$$h(\varphi, \theta) = h(\theta, \varphi).$$

**Exercise 2 (3 Points).** Let $(V \otimes V)_s$ be the space of symmetric tensors:

$$(V \otimes V)_s = \{ h \in V \otimes V | Sh = h \}.$$ 

Show, that this space is a vector space, give a basis (in terms of a basis $e_1, \ldots, e_n$ of $V$) and determine its dimension.

**Exercise 3 (3 Points).** Let $a^i_j, b^i_j \in \mathbb{R}$ be the uniquely defined coefficients, such that 

$$e_j = a^i_j \tilde{e}_i$$

$$e^j = b^i_j \tilde{e}^i$$

holds for any $j \in \{1, \ldots, n\}$. Show, that $a^k_j b^j_i = \delta^i_j$ holds for any $i, j \in \{1, \ldots, n\}$.

Let 

$$h = h^{ij} e_i \otimes e_j = \tilde{h}^{ij} \tilde{e}_i \otimes \tilde{e}_j \in V \otimes V$$

be a tensor. Show that 

$$\tilde{h}^{kl} = h^{ij} a^k_i a^l_j$$

holds for any $k, l \in \{1, \ldots, n\}$. 

**Exercise 4 (3 Points).** Show, that the tensors $h = e_i \otimes e^i$ and $\tilde{e}_i \otimes \tilde{e}^i$ in $V \otimes V^*$ coincide. What element of $B(V)$ does $h$ correspond to under the isomorphism $\Phi$ from Proposition 2.14.

The written solutions to these exercises should be handed in immediately before the lecture on Monday the 11.06.2018.