Abstract

The goal of this thesis is the development of a fast and robust algorithm that is able to detect patterns in flow fields independent from their orientation and adequately visualize the results for a human user.

Figure 1 Visualization of the occurrences of two counter oriented vortices in a 2D simulation of the von-Kármán vortex street with moment invariants and line integral convolution.

This thesis is an interdisciplinary work in the field of vector field visualization and the field of pattern recognition.

A vector field can be best imagined as an area or a volume containing a lot of arrows. The direction of the arrow describes the direction of a flow or force at the point where it starts and the length its velocity or strength. This builds a bridge to vector field visualization, because drawing these arrows is one of the fundamental techniques to illustrate a vector field. The main challenge of vector field visualization is to decide which of them should be drawn. If you do not draw enough arrows, you may miss the feature you are interested in. If you draw too many arrows, your image will be black all over.

We assume that the user is interested in a certain feature of the vector field: a certain pattern. To prevent clutter and occlusion of the interesting parts, we first look for this pattern and then apply a visualization that emphasizes its occurrences. In general, the user wants to find all instances of the interesting pattern, no matter if they are smaller or bigger, weaker or stronger or oriented in some other direction than his reference input pattern. But looking for all these transformed versions would take far too long. That is why, we look for an algorithm that detects the occurrences of the pattern independent from these transformations.
In this thesis, we pursue two approaches to tackle this goal: the Clifford algebras and the moment invariants.

The Clifford algebras are a generalization of the complex numbers to hypercomplex numbers. They come along with the geometric product. The geometric product of two vectors is a rotor, that means, a representation of a rotation that contains information about their relative alignment. We try to exploit this property in the first part of this thesis. We were able to apply it to the detection of rotational distortions in the color space of color images, but not to the detection of misalignments in 3D flow fields, because they behave differently.

In the second part of this thesis, we work with moment invariants. Moments are the projections of a function to a function space basis. In order to compare the functions, it is sufficient to compare their moments. Normalization is the act of transforming a function into a predefined standard position. Moment invariants are characteristic numbers like fingerprints that are constructed from moments and do not change under certain transformations. They can be produced by normalization, because if all the functions are in one standard position, their prior position has no influence on their normalized moments.

With this technique, we were able to solve the pattern detection task for 2D and 3D flow fields by mathematically proving the invariance of the moments with respect to translation, rotation, and scaling. In practical applications, this invariance is disturbed by the discretization. We applied our method to several analytic and real world data sets and showed that it works on discrete fields in a robust way. Examples of the corresponding visualizations are depicted in Figures 5 and 6.