High temperature expansions for disordered systems

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- Potts model & disorder
- Why series expansions?
- Linked cluster & star graph expansion
- Selected results

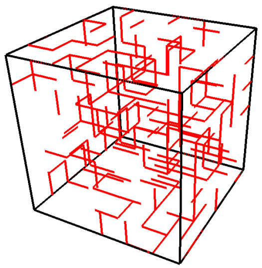
Start

q-state Potts model with quenched disorder

$$\mathcal{Z}[\{J_{ij}\}] = \mathrm{Tr}\,\exp\left(-\beta \sum_{<ij>} J_{ij}\delta(S_i,S_j)\right)$$

$$\propto \sum_{\text{cluster } C} q^{c(C)} \prod_{<ij>\in C} J_{ij}$$

- random couplings: J_{ij} choosen according to some probability distribution $P(J)$
  - spin glasses $J = +1/-1$
  - random bond models $J = +1/+5$
  - correlated disorder
- geometric disorder: bond dilution, site dilution



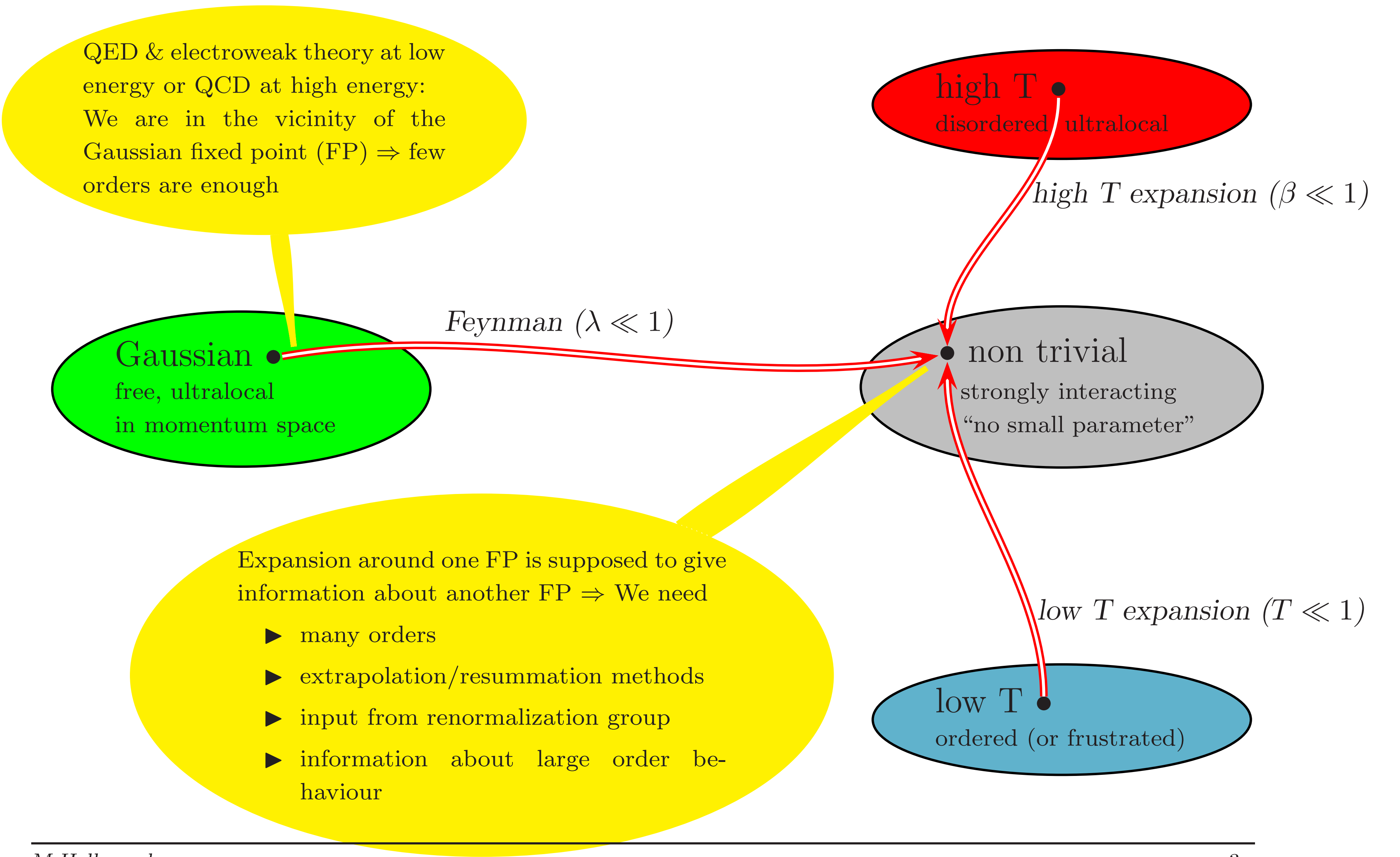
Effect of disorder on critical behaviour:

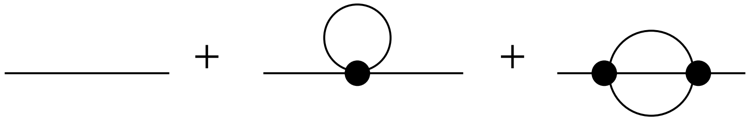
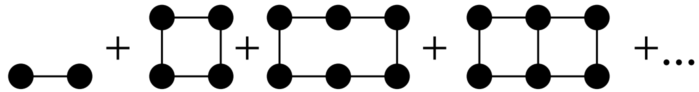
- irrelevant $\longrightarrow$ only non-universal quantities (T_C) change
- relevant $\longrightarrow$ new fixed point
- marginal $\longrightarrow$ log corrections

or worse: Aharony 1976 4D Ising model with disorder

$$\chi \propto \xi^2 \propto t^{-1}|\log t|^{\frac{1}{3}} \quad \xrightarrow{\text{disorder}} \quad \chi \propto \xi^2 \propto t^{-1}\exp\left[\left(\frac{6}{53}|\log t|\right)^{\frac{1}{2}}\right]$$

Renormalization group fixed points and expansions



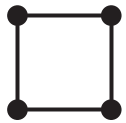
	weak coupling expansion	high T expansion
starting point:	decoupled d.o.f. in p-space	decoupled d.o.f. in x-space
	Feynman graphs 	lattice graphs 
difficult part:	$\int dp_1 \int dp_2 \ldots \int dp_n$ momentum integral for each line	$\sum_{x_1} \sum_{x_2} \ldots \sum_{x_m} 1$ lattice sum for each vertex
constraint:	$\sum_{\text{Vertex}} p_i = 0$	$\langle ij \rangle$ neighbours on graph $\Rightarrow$ neighbours on lattice

Embedding numbers

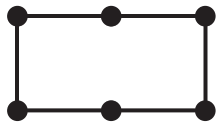
- number of maps Graph $G \to$ Lattice Λ with fixed base point which map adjacent vertices of G to nearest neighbours of Λ.
- **Weak embedding number:** different graph vertices $\to$ different points on Λ
- **Free embedding number:** different graph vertices may be mapped to the same lattice point

Examples for weak embedding numbers in $\mathbb{Z}^d$

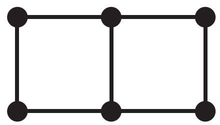
$$d$$



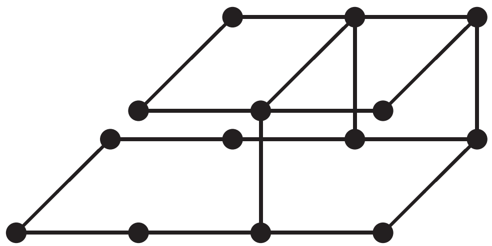
$$\binom{d}{2}$$



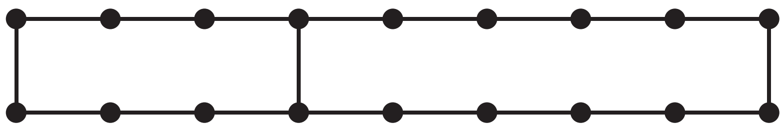
$$2\binom{d}{2} + 16\binom{d}{3}$$



$$2\binom{d}{2} + 12\binom{d}{3}$$



$$12048\binom{d}{3} + 396672\binom{d}{4} + 2127360\binom{d}{5} + 2488320\binom{d}{6}$$



$$8\binom{d}{2} + 275184\binom{d}{3} + 18763392\binom{d}{4} + 208611840\binom{d}{5} + 645442560\binom{d}{6} + 559964160\binom{d}{7}$$

- d-dependency is explicit

Linked cluster vs Star graph expansion

linked cluster expansion

- connected graphs w. multi-edges
- free embedding numbers
- contribution of graph is simple

star graph expansion

- 1-vertex-irreducible graphs, no multi-edges
- weak embedding numbers
- need Z and $(< s_i s_j >)_{ij}$ for every graph
- only some quantities — susceptibility, free energy (but not second moment) — have star graph expansion
- application to quenched disorder possible

pure 3D Ising model:

Sykes et al.	1973	13. order
Nickel, Rehr	1990	21. order
Butera, Comi	2000	23. order
Campostrini et al.	2000	25. order

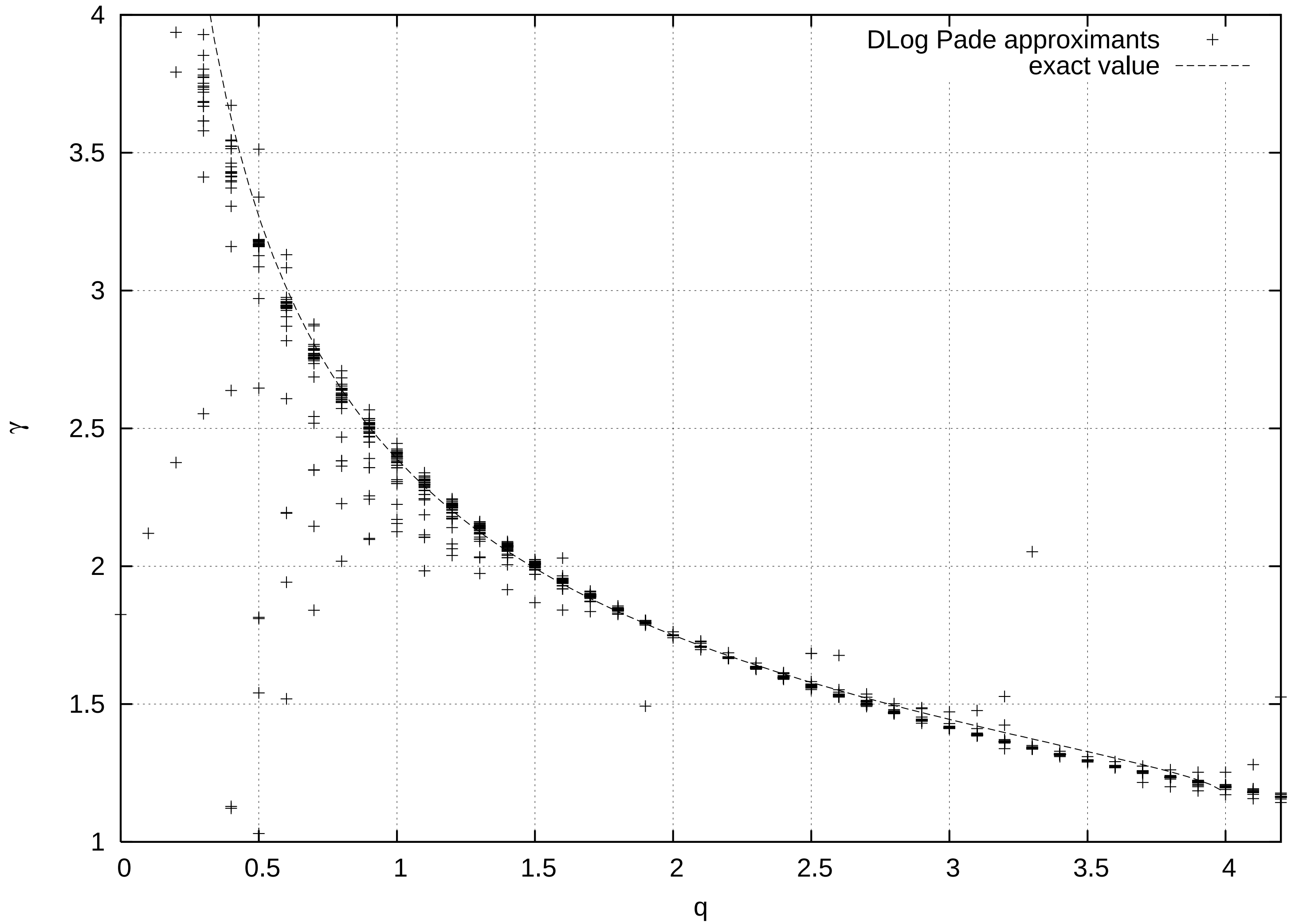
disordered models:

Singh, Chakravarty	1987	15	3D Ising glass
Schreider, Reger	1993	10	q-Potts glass
Roder, Adler, Janke	1998	11	2D RB Ising
Hellmund, Janke	2001	17	bond-diluted Potts, all D
Hellmund, Janke	2004	19	bond-diluted Potts, $D < 6$

Example: pure q-state Potts model

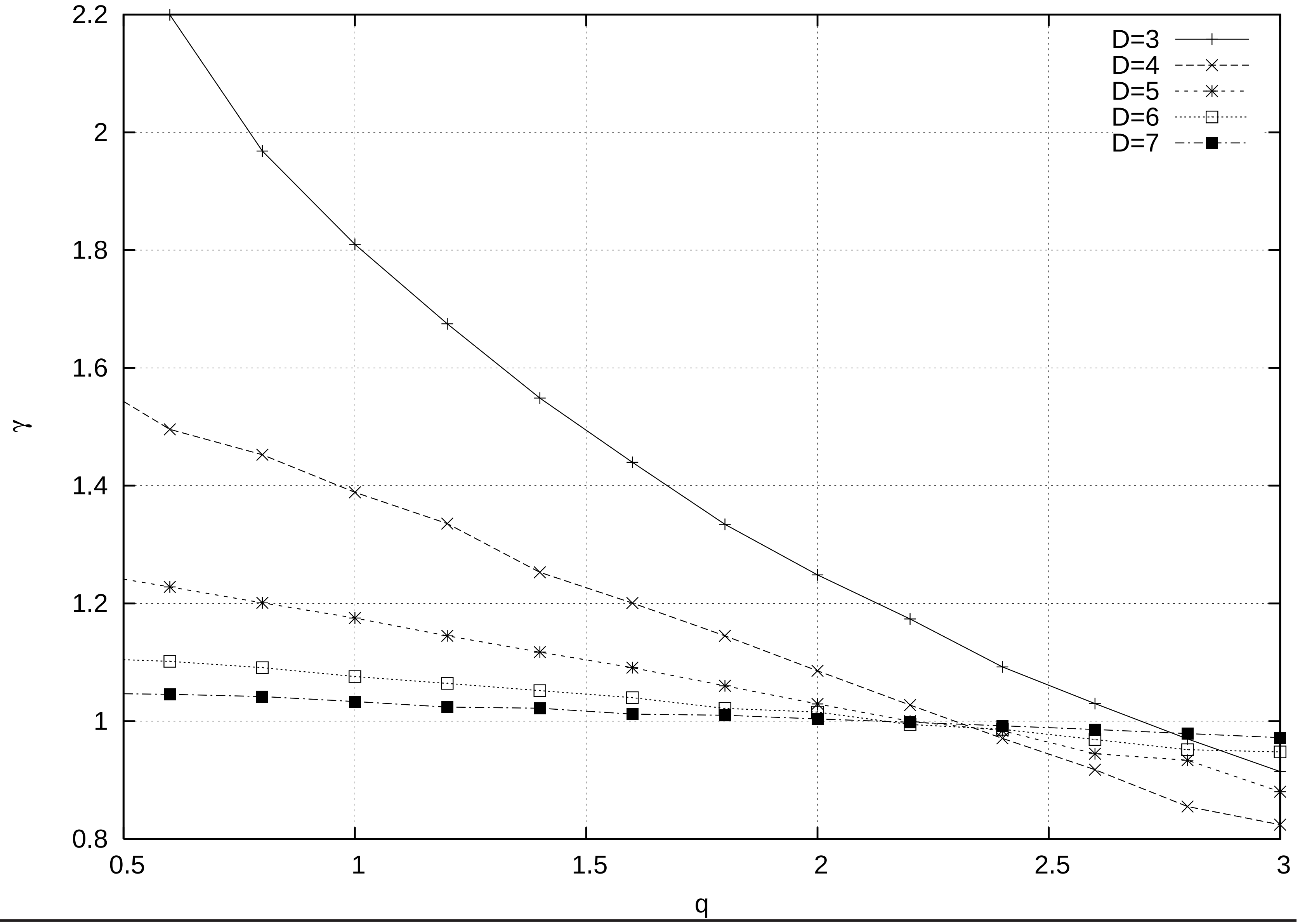
- $D = 2$: exact behaviour known from CFT — 2. order PT for $0 \leq q \leq 4$

$$\gamma = \frac{4 + 3k^2}{12k - 6k^2} \quad \text{where} \quad \sqrt{q} = -2\cos\frac{\pi}{k}.$$



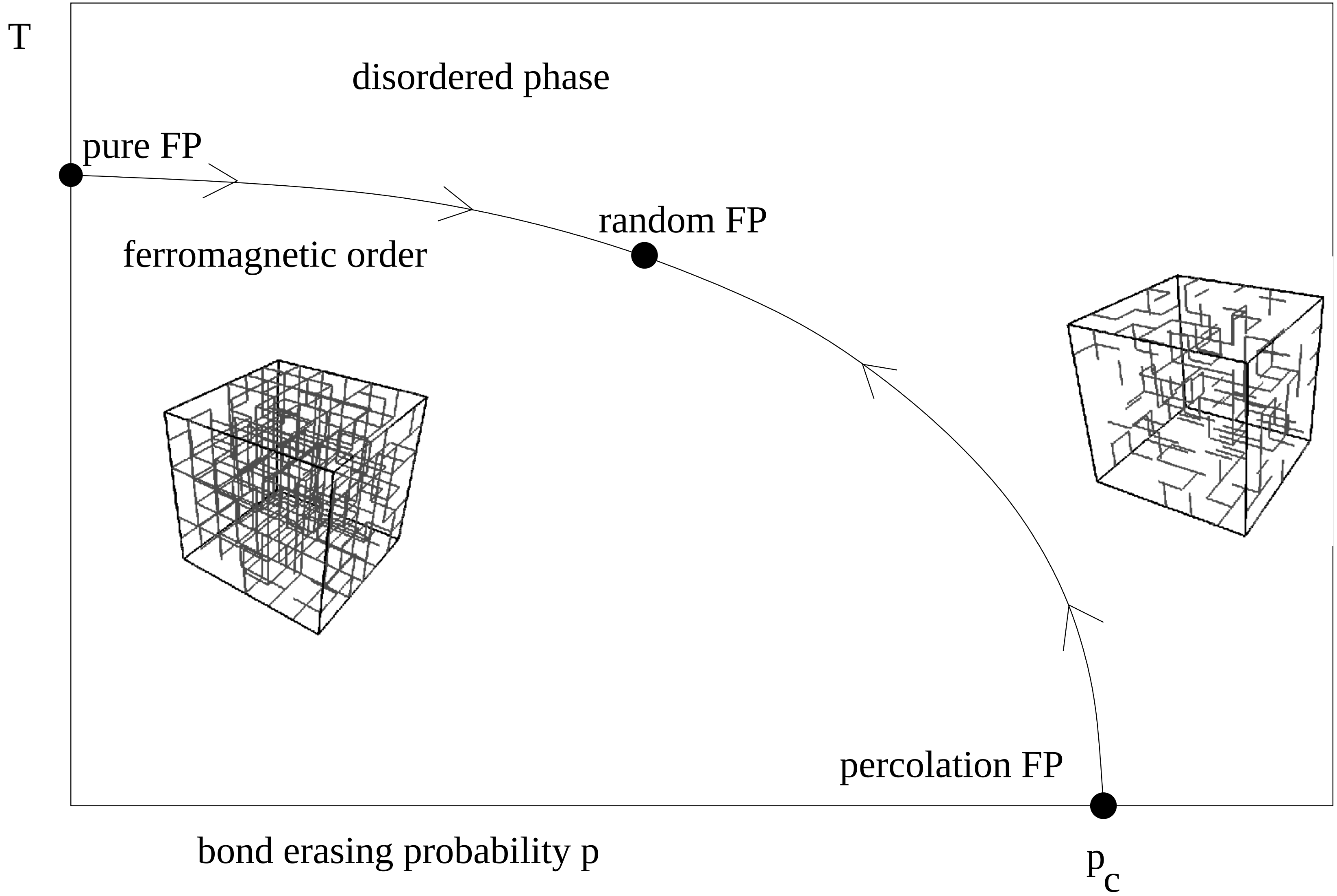
Example: pure q-state Potts model

- $D > 2$: 2. order PT for $q = 2$; 1. order for larger q



Example: Bond-diluted Ising model in 3D

$$\chi(p,\beta) = 1 + 6p\beta + 30p^2\beta^2 + 150p^3\beta^3 + \ldots \text{ 127 terms} \ldots + 10398318680694p^{19}\beta^{19} + O(\beta^{20})$$



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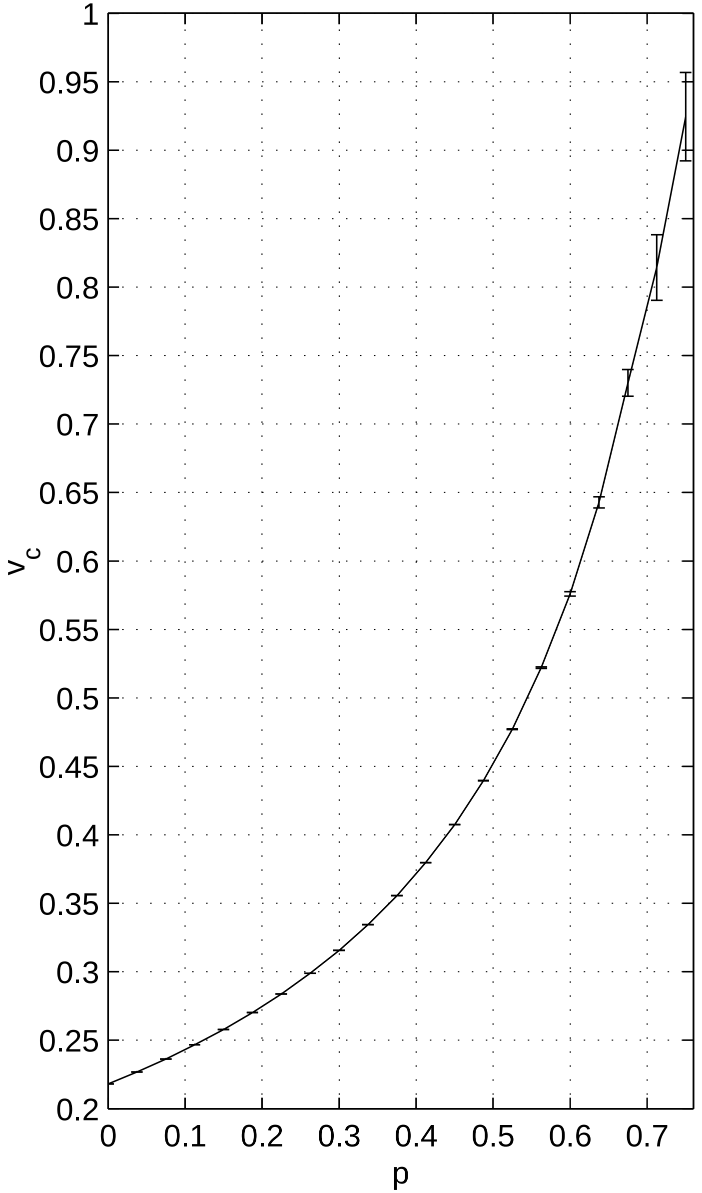

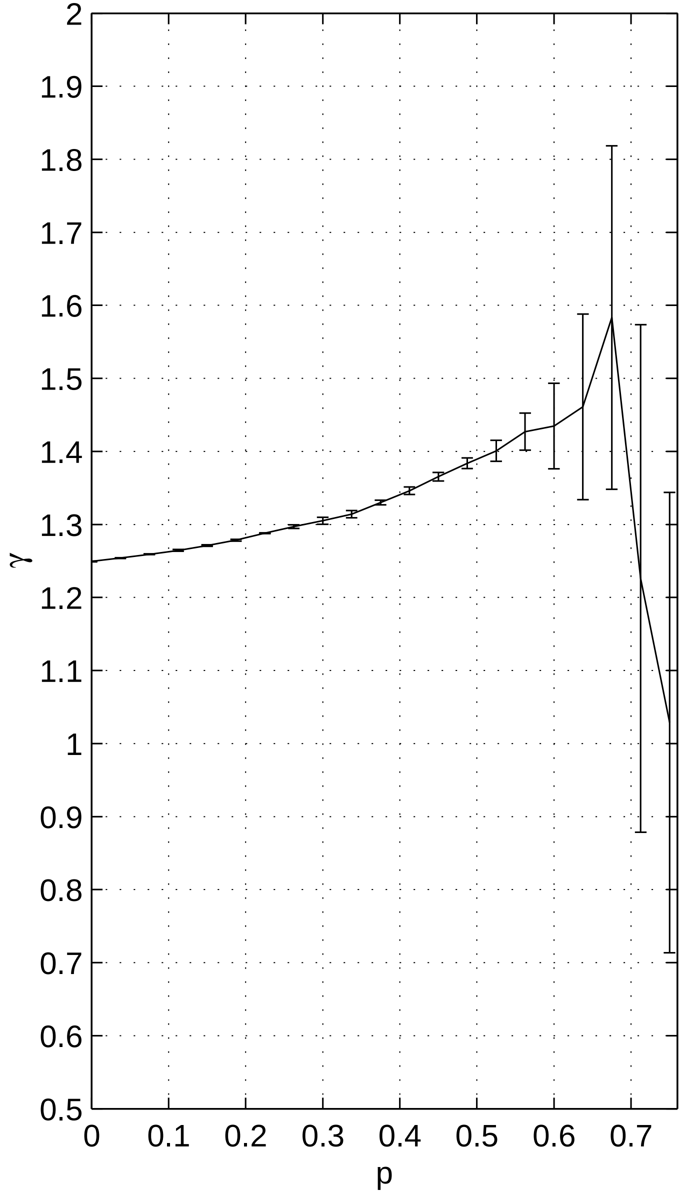

Conclusions

- Series expansion techniques are an interesting mixture of higher combinatorics, graph theory, computer algebra . . .
- There is room for new ideas/algorithms.
- Need better series analysis/extrapolation techniques for crossover.
- Quenched disorder is hard.