Seminar Topology: Topics

Block 1: Tychonoff's theorem, Baire spaces and more (3-5 persons)

1) Tychonoff's theorem (1-2 pers.) - [CR] (in German), [JT]

2) Baire spaces and topological games (1-2 pers.) - [K-book, Sections 8.A-E,H]

3) A typical continuous function is nowhere differentiable (1 pers.) – [Thim, Sec. 4.2]

4) A typical contraction is unitary (1 pers.) – [E]

Block 2: Ultrafilters (5-6 persons)

5) Ultrafilters as a topological space and the Stone-Cech compactification (2 pers.) - [JK, Sect. 1.2]

6) Algebraic structure of filters and ultrafilters (1 pers.) - [JK, Sec. 1.3 until Cor. 1.61]

7) Structure of semitopological semigroups (1-2 pers.) - [JK, Sect. 1.3 after Cor. 1.61]

8) Ultrafilters via finitely additive measures (1 pers.) - [JK, Sec. 1.4], might be extended by topics above if too few material

Block 3: Applications to Ramsey theory and voting (5-6 persons)

9) Application of ultrafilters to fair voting: Arrow's theorem (1 pers.) - [JK, Sec.]

10) Ramsey's theorem on colouring of graphs (1-2 pers.) – [JK, Sec. 2.1]

11) Hindman's theorem via ultrafilters (1 pers.) - [LG], [JK, Sec. 2.1]

12*) Van der Waerden theorem (1 pers.) - [JK, Sec. 2.3]

13*) Hales-Jewett theorem (1 pers.) - [JK, Sec. 2.3]

Literature

[K-book] Alexander Kechris, „Descriptive set theory”, Springer-Verlag, 1995.

[JK] Jakub Konieczny „Applications of Ultrafilters in Ergodic Theory and Combinatorial Number Theory”, Master thesis, University of Amsterdam, 2013. Available under https://jakubmichalkonieczny.wordpress.com/publications/

[CR] Claudius Röhl „Das Auswahlaxiom”, Diplomarbeit, Universität Leipzig, 2017.

[LG] Leo Goldmacher, „Hindman's theorem via ultrafilter“, notes, available at https://web.williams.edu/Mathematics/lg5/Hindman.pdf

[JT] John Terilla, „Tychonoff's theorem”, notes, 2010. Available at https://math.hunter.cuny.edu/mbenders/notes4.pdf.

[Thim] Johan Thim, „Continuous nowhere differentiable functions”, Master thesis, Lulea University of Technology, 2003. Available at https://ltu.diva-portal.org/smash/get/diva2:1022983/FULLTEXT01.

[E] T. Eisner, „A typical contraction is unitary”, Enseign. Math. (2) 56 (2010), 403-410. Available at https://arxiv.org/pdf/0807.1994.